Bildung—Then And Now In Danish High School And University Teaching And How To Integrate Bildung Into Modern University Teaching

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ABSTRACT

In the history of mankind three important philosophical and scientific revolutions have taken place. The first of these revolutions was the mathematical-axiomatic revolution in ancient Greece, when the philosophers from Thales of Miletus to Archimedes built up the abstract deductive method used in pure mathematics. The second took place in the Renaissance when the experimental-inductive method was introduced by the British philosopher Francis Bacon (1561–1626), the Danish astronomer Tycho Brahe (1546–1601) and the Italian physicist Galileo Galilei (1564–1642). This method was fully completed to the hypothetical-deductive method by the English mathematician and physicist Isaac Newton (1642–1727) in his famous book "Philosophiae Naturalis Principia Mathematica" from 1687 and extended to other natural sciences by several researchers such as the naturalists Niels Stensen (1638–1686) from Denmark, John Ray (1627–1705) from England, and Carl von Linné (1707 - 1778) from Sweden (Curtler 2003, Huxley [21] 2007). The third philosophical and scientific revolution was a consequence of the Enlightenment period in the 18th century. It is, what we might call, the educational revolution where new didactic and pedagogical means and aims were introduced in primarily higher education. This revolution took place in the first half of the 19th century and it still has a remarkable impact on high school and university teaching in many countries.

The main ingredient of the educational revolution was the concept of "Bildung" which was originally introduced by several German philosophers especially Immanuel Kant (1724–1804) (Kant [26] 1784, [27] 1787, [28] 1788, [29] 1790, [30] 1798, and [31] 1803) and the Prussian philologist, lawyer, and politician Wilhelm von Humboldt (1767–1835) (Humboldt 1792, Bohlin [1] 2008). In Denmark "Bildung" is called "Dannelse", in Sweden they say "Bildning" but in English there is no specific word for this concept. Although we, with full right, might use the nouns "Education" or "Enlightenment" for "Bildung" none of these words are fully covering the concept, so we choose to say "Bildung" also in an English text.

In this paper we, first of all, will try to find a useful and definite definition of Bildung and we will consider Bildung as a pedagogical and didactic principle that has had a strong influence on teaching at high schools and universities during the last 200 years. From 1810 "Bildung" was a key concept in German university teaching and education where the main purposes were to give the students: (1) advanced teaching based on research, (2) ability to carry out scientific research on their own, and (3) a large amount of scientific and philosophical knowledge within all academic disciplines such that they could act with dignity as members of the learned and academic society. (Flexner 1930, Huxley [22] 1876, Jaspers 1923, 1946).

Also in Denmark and the other Nordic countries Bildung has played a very central role in high school and university teaching since the middle of the 19th century. Bildung as a didactic principle and useful educational tool was fully introduced in Denmark in 1848 where the great Danish philologist Nicolai Madvig (1804–1886) accomplished an extensive reform of the teaching at the University of Copenhagen. The faculties were reorganized and new disciplines such as economics were introduced. (Korsgaard 2004, Boserup 1992, Christensen [4] and [5] 2009, Krarup 1955, Petersen [52] 1993, Slagstad 2003, Oersted 1850).

During many years Bildung was a quite natural educational principle and also an aim on itself the all high school and university teaching not only in Denmark, Norway and Sweden but also in most western countries. However, after 1970 the classical concept of Buildung was weakened as a didactic and pedagogical principle, first slowly but since 2000 with a much faster speed, such that Bildung, in most high school and university teaching, didn't play a central role any longer, neither in Denmark nor in many other countries, especially in other the Scandinavian countries. The main reason for this development is that the syllabuses and the teaching levels were decreased considerably to get more students to graduate from high schools and universities. It was pointed out from the teaching authorities that for economical reasons students had to graduate faster than they had done previously and that the usual academic level was superfluous for most students. The result of this non academic attitude was a mass production of students and candidates with poor and fragmented knowledge (Lars Løvie [55] 2003, Olesen [42] 2007, Olesen [43], [44], and [45] 2008, Olesen [46], [47], and [48] 2009). Also, both in Denmark, Norway and Sweden, the drop-out rates and the rates of failure at the universities were increasing in a disastrous way (Olesen [43], [44], and [45] 2008, Olesen [46], 47], and [48] 2009).

However, changing the basic university teaching of mathematics at The Department of Economics at the University of Copenhagen since 2008, such that elements of Bildung were incorporated into the lectures, a remarkable impact was observed. The rate of failure dropped considerably and the students became more engaged and obtained a better ability to do their studies and to work concentrated on their own (Olesen [43], [44], and [45] 2008, Olesen [46], 47], and [48] 2009).

In this paper, after having looked at the historical development, we will concentrate on new ways in which Bildung might be developed and how Bildung might work in modern university teaching in the near future.

INTRODUCTION

In pedagogical and didactic literature one usually says that Bildung was defined by the German philosopher Immanuel Kant in 1784 (Kant [26] 1784, Slagstad 2003). In his famous article on being enlightened and educated he emphasized that "a person is enlightened when this person leaves his or her self-inflected incapability of managing own affairs". And incapability of managing one's own affaires means "lack of using one's intellect without guidance from somebody else". Hence, when a human being has been enlightened in this definite respect he or she has got Bildung. So in this way the concept of Bildung is now well defined. Or as Kant also stated this shortly in Latin: "Sapere aude", i. e. "Dare to know".

Bildung became both an axiom and an aim in a new pedagogical philosophy. But as the word Bildung indicates it has a religious background coming from the "Imago Dei" in early Christian mysticism. Imago Dei means God's image, so the concept of Bildung is referring to an action or a process in which something is depicted. That is: What we experience and perceive is depicted in our minds.

Immanuel Kant was born in Königsberg in East Prussia and his parents were deeply religious and pietists (Hartnack [15] 1966). Of course this had a big influence on Kant's way of thinking and it is interesting to see that Bildung had its origin in the pietistic movement. Also he was influenced pretty much by his university teacher, the natural philosopher Martin

Knutzen (1713–1751), who introduced him to Newton's natural philosophy and mechanical physics.

To a pietist the virtues of duty and industry are very central and important, and to do one's duty and being industrious it is necessary to be educated. Therefore to the pietistic philosophers it was very important to educate people such that they were able to read and write and to do their work conscientiously. Hence the pietistic movement was also a pedagogical movement with educational aims.

The pietistic school of philosophy was started by the German theologians Philipp Jacob Spener (1635–1705) and August Hermann Francke (1663–1727) and both its theological and pedagogical ideas were spread in the beginning of the 18th century to the united monarchy Denmark-Norway and to Sweden (Korsgaard 2004, Lange 2003). It is interesting to see that to the first pietistic king of Denmark-Norway, Frederik IV (king 1699–1730), it was important to establish schools for children all over the united monarchy such that his subjects learned to read and write. It is also interesting to see that pietism led to a new and deep respect to the individual. When the Danish-Norwegian clergyman Hans Egede (1686–1758) came to Greenland in 1721 with the purpose to Christian the population he denied baptizing anybody until they had got a sufficient education (Kristensen 2008, Høiris and Fægteborg 2009). So in fact Hans Egede and the pietistic philosophers had the idea of Bildung as an important educational and civilizing aim.

Since Kant was brought up in a pietistic home it is no wonder that his pedagogical philosophy was based on the pietistic virtues. It is also important to notice that Kant was influenced by the English philosopher John Locke (1632–1704) and the French philosopher Jean-Jacques Rousseau (1712- 1778). In 1690 John Locke published his great work "An Essay Concerning Human Understanding" (Hartnack [14] 1965), and he pointed out that when a child is born its mind is totally empty, "tabula rasa" (a blank slate or a clean blackboard). But nature outside the child is full of light, and by means of experience and recognition the human mind is enlightened. So, according to John Locke, our knowledge is determined only by experience and is derived from our sense perception.

This point of view was also central to Rousseau when he wrote his pedagogical masterpiece "Émile ou de l'éducation" in 1772 (Korsgaard [55] 2003, Korsgaard [35] 2004). But to Rousseau a child had to be brought up and to be taught or rather guided on its own personal conditions to become an educated person and a free citizen in a given political community. Such a person is an individual of the people considered as "demos".

To Kant, being influenced by pietism, both Locke's and Rousseau's thoughts were canalized into his definition of an enlightened person (Kant [26] 1784), and to reach the aim of giving Bildung and education to individuals he had the opinion that upbringing and education of children and young people must be carried out using firm discipline. Duty and industry, the pietistic virtues, were clearly central in all education according to Kant.

The word Bildung was used already in 1769 by the German philosopher Johann Gottfried von Herder (1744–1803) when he published his book "Journal meiner Reise" (Bohlin [1] 2008, Korsgaard [55] 2003, Korsgaard [35] 2004). Here "Imagio Dei" has got a quite new interpretation: Since man is created in the image of God the meaning of the human life is to accomplish a divine aptitude. Herder calls this divine aptitude "Humanity". A human being should no longer be formed in the image of God but in the image of mankind. Hence Bildung

is the very aim of educating the individual. In this context, according to Herder, language is the most important thing because your mother tongue is a gift from God and a tool for reasoning. In fact our language determines our thoughts. The relationship between language and human reason was later described detailed by Herder in "Abhandlung über den Ursprung der Sprache" from 1772 (Stjernfelt 2008). What binds a people together is exactly the mother tongue according to Herder, and hence he doesn't consider a people in the meaning of "demos" (a people as a political unity), such as Rousseau and Kant had done, but in the meaning of "ethnos" (a people as an ethnic unity) (Korsgaard [55] 2003, Korsgaard [35] 2004).

In his work "Theorie der Bildung des Menschen" from 1792 Wilhelm von Humboldt developed his theory of Bildung (Humboldt 1792), and he had a pedagogical point of view close to Kant's but he was an adherent of Herder's opinion that language is important for human understanding. This point of view was shared by another German philosopher of that time, Johann Gottlieb Fichte (1762–1814), who also pointed out that an educated people with a common language constituted the concept of a nation. In this sense Bildung was a prerequisite for a national identity (Gross 2006, Stjernfelt 2008) and he radicalized Kant's philosophy in his work "Grundlagen der gesammten Wissenschaftslehre" published in 1794. And a little later, at the beginning of the 19th century, the German philosopher Friedrich Schleiermacher (1768–1834) who established modern hermeneutics as a general scientific discipline said that "understanding a text is the capability to reproduce its creational process" (Stjernfelt 2008). This might be considered as a very interesting imperative of pedagogical Bildung.

In his work "Über Pädagogik" Kant pointed out that nature of man should not be considered as an in advance given resistance against education that had to be defeated but rather a basis and a condition for personal and educational development (Kant [31] 1803). But since developing a person's natural aptitude doesn't happen on its own all upbringing and education is a specific art or science where the individual is respected and guided by skilled teachers. This was a specific sort of humanism that got great importance when Bildung later was implemented into high school and university teaching in several European countries.

Both to Kant and many other philosophers of his time enlightenment and Bildung should not only be considered in relation to the individual but also as a cultural, social, and political community with great importance to mankind in full. This point of view is the fundament of what we might call "the educational theoretical trinity": (1) Bildung is first of all an aim for all individuals and it is closely related to concepts such as duty, freedom, emancipation, autonomy, individualism, responsibility, reason, and knowledge. (2) The relationship between the individual and the whole world, i. e. knowledge about nature and the whole universe, tolerance and respect for other people, humanism, and objectivity. (3) The relationship between the individual and society. How does one behave to be a responsible citizen in a given political unity? This is a question concerning political morality. To Rousseau the answer of this question was the so-called "social treaty", to Kant the answer was citizenship. To cut it short: Bildung is the relationship between the educated self, society, and the world. Furthermore, "the educational theoretical trinity" is still central in modern didactic and pedagogical philosophy (Korsgaaard [55] 2003, Korsgaard [35] 2004).

ELEMENTS OF KANT'S PHILOSOPHY WITH RESPECT TO BILDUNG

Among all the philosophical works Kant wrote, especially three have had a central impact on our understanding of Bildung. These three are his famous "critiques": "Kritik der reinen Vernunft", Eng. "Critique of Pure Reason", (Kant [27] 1781, Kant [27] 1787), "Kritik der praktischen Vernunft", Eng. "Critique of Practical Reason", (Kant [28] 1788), and "Kritik der Urteilskraft", Eng: "Critique of Judgment", (Kant [29] 1790). In "Kritik der reinen Vernunft" he outlines his theory of knowledge, in "Kritik der praktischen Vernunft" he deals with morality and the human will, and in "Kritik der Urteilskraft" Kant establishes a theory of human judgment.

Kant's theory of knowledge is based upon the two so-called "forms of sensibility" or "forms of sensible intuition": Space and time. Both space and time are given a priori prerequisites (i. e. they are given prior to all experience) and they are necessary to all understanding. To perceive a phenomenon outside ourselves we must make our observations and experiences in relation to space and time. Next to be conscious of what we have observed we need "the forms of reason" and also, the forms of reason are given a priori to our minds as necessary conditions for any possible perception. The forms of reason can be categorized in four different categories of understanding: Quantity (unity, plurality, and totality), quality (reality, negation and limitation), modality (existence-non existence, possibility-impossibility, and necessity-contingency), and relation (inherence-subsistence, e. g. substance-subject, causality-dependence, e. g. cause-effect, and interaction, e. g. reciprocity between action and reaction) (Hartnack [15] 1966).

To Kant there are two different sorts of judgments or propositions: The analytic and the synthetic. In an analytic proposition the subject (the thing we want to tell something about) includes the predicate (the property of the subject). Hence an analytic proposition is given a prior. For example "All mammals are animals". But in a synthetic proposition the predicate and the subject are disjoint. For example: "All stones have weight". In the concept "stone", the concept "weight" is not included. Therefore a proposition which is a posteriori, i. e. not a priori and therefore based upon experience, must be synthetic.

The empiricists and rationalists before Kant assumed that all synthetic statements required experience to be perceived. However, Kant claimed that there also exist synthetic propositions that are a priori, and he himself gave the example "5 + 7 = 12" (Kant [27] 1787, Hartnack [15] 1966, Olesen [41] 2007). At first glance, when we have defined addition, subtraction, multiplication and other mathematical operations, mathematical results seem to be analytic. But this is not true Kant says in the preface of the second edition of his "Kritik der reinen Vernunft". If the numbers 5 and 7 in the calculation 5 + 7 = 12 are examined, there is nothing in them by which the number 12 can be inferred. Hence 5 + 7 = 12 is not an analytic proposition, and 5 + 7 = 12 tells us something new about the world that is not based upon experience, thus it is synthetic. It is self-evident and undeniable a priori, but, as we have just noticed, it is synthetic. In this way Kant tells us that mathematics is an abstract and deductive science in which the theorems are giving us new knowledge, but they are not, in any respect, related to experience.

Of course Kant's theory of knowledge is important to the concept of Bildung, but that is his theory of morality as well. Here his categorical imperative is very central: "Act only in such a way that you want all others to act in the same situation" (Kant [28] 1788, Hartnack [15] 1966). The categorical imperative is a principle, that is intrinsically valid, it is good in and of

itself, and must be obeyed by everybody in all situations and circumstances. Here we see the heritage Kant had from pietism where duty is an important imperative that is beyond dispute.

The ability of judgment was, according to Kant, another important ingredient of being educated and in his theory the aesthetical experience is a delight that is based upon reactions which course a harmonic relationship between the different abilities of our understanding (Kant [29] 1790, Koch [34] 2004). Hence knowledge, ethics, and judgment constitute a new trinity of Bildung in Kant's philosophy, a trinity that became very significant to later philosophers of didactics and pedagogy (Korsgaard [35] 2004, Christensen [6] 2009). Furthermore, this trinity, as we shall see at the end of this paper, will also play a central role as an aim of the way Bildung is applied to modern university teaching.

To Kant it was very important to obtain a new and quite different organization of the universities. The old scholastic Aristotelian universities had not been changed since they were established in The Middle Ages and originally they were primarily teaching institutions in theology and controlled by the Catholic Church or after the Reformation in the protestant countries by the king and his administration. The faculties were ranged such that the theological faculty was the most prominent, followed by the law faculty, the medical faculty, and at last, at the very bottom, the philosophical faculty, that served all the others, was placed. The new scientific revolution which took place during The Renaissance implied that research in mathematics, physics, anatomy and other natural sciences became very significant, but to place such research at the old universities was quite difficult and hence many so-called free scientific academies where research of the natural sciences could be carried out were established in many countries, e. g. Royal Society of London, Berlin Academy, and Petersburg Academy. Also some new secular universities were established, e. g. University of Göttingen in 1734. But as a consequence of the educational revolution at the end of the 18th century Kant and Wilhelm von Humboldt worked out new ideas concerning university teaching, university research and university organization.

To Wilhelm von Humboldt it was very important to build up a new university organization where teaching and research were strongly linked to each other and where the students were "free academic citizens". The studies should be based upon the New-Humanistic movement where the basic point was knowledge about Greek culture and Greek philosophy. On this basis teaching in modern sciences must be built and the didactic methods and aims for all university studies were strongly influenced by the new theory of Bildung (Humboldt 1792). Also Kant contributed to this new policy of universities. In 1798 he published his book "Der Streit der Fakultäten" (Kant [30] 1798) where he argued for a total reorganization of the universities such that the philosophical faculty became the most important because in this faculty all the sciences and philosophical subjects were studied and developed. Based upon these modern ideas and principles Humboldt established the new (and soon very famous) Berlin University in 1810. Now, after World War II, it has been renamed "The Humboldt University" in honor of its founder.

THE PHILANTHROPIC BILDUNG

Until 1809 the high school teaching in the united monarchy Denmark-Norway prepared the students (called disciples) to study at the Danish universities in Copenhagen and Kiel. The high schools were so-called Latin schools since the main aim was to teach the disciples the Latin language and rhetoric, logic, elementary mathematics, astronomy and music. These were the old well known scholastic disciplines "Trividum" and "Quadrividum". The students had to pass an exam called "Artium" at the universities to be matriculated. And then the first

year of university studies were mainly old philosophical subjects. In fact the students didn't learn anything about the new natural sciences (e. g. infinitesimal calculus and Newtonian mechanics) and that was an increasing problem to the Danish-Norwegian society at the end of the 18th century when enlightenment just had its full breakthrough. Big reforms of the academic virtues and the teaching at the high schools and the universities were strongly imperative and in 1790 a royal commission was appointed under the guidance of Duke Frederik Christian of Augustenborg (1765–1814) (Korsgaard [35] 2004). The duke was brother-in-law to Crown Prince Frederik, later King Frederik VI.

At that time some new ideas about education had come to the Danish-Norwegian monarchy. Not only Rousseau's revolutionary teaching philosophy influenced the quite open debate but especially the so-called philanthropic school was of great importance to some new educational attempts that were carried out both at the Academy of Soroe and later at the University of Copenhagen. The idea of the philanthropic school was that all education should take its origin in experimental and sensible facts (e. g. the natural sciences and crafts) and give practical knowledge to the students such that they were able to work usefully as citizens and be happy human beings. This ideal was quite new but with heavy momentum at a time where society changed considerably (Korsgaard [35] 2004). The pedagogical tool was mainly the old Socratic and the philanthropists were clearly influenced by Rousseau.

The first prominent philanthropist was the German educationalist Johann Bernhard Basedow (1724–1790) who worked at the Danish Academy of Soroe and later in Altona near Hamburg and in Dessau where he founded his own philanthropic school. Hence his pedagogical ideas and educational philosophy, that were outlined in his great work "Vorstellung an Menschenfreunde und vermögende Männer über Schulen, mit einem Plane eines Elementarbuchs der menschlichen Erkentniss" from 1768, influenced Danish education pretty much in the late 18th century (Koch [33] 2003, Korsgaard [35] 2004, Lange 2003).

While the educational royal commission was still working the philanthropic pedagogical ideas were introduced at the University of Copenhagen. At that time the education of high school teachers for the Latin schools were considered a growing problem. The teachers were usually young theologians or philologists and none of them could meet the new requirements to teach in the natural sciences or new mathematics. Therefore it was determined to establish a new faculty of education at the University of Copenhagen from which all high school teachers should graduate. This faculty must offer four different studies: (1) A philological line, (2) A line with religion and anthropology, (3) A geographical-historical line, and (4) A line with mathematics, physics and other natural sciences. But furthermore, in the teaching of this faculty didactics and problem oriented educational questions had to be included. Actually, these requirements were influenced of the ideas of Bildung and they were very modern—also seen with today's eyes. (Boserup 1992).

To carry out these new initiatives the royal commission wanted to employ the famous German philologist and philosopher Christian Gottlob Heyne (1729–1811), but he would rather stay in Göttingen (Groos 2006, Boserup 1992). I stead of him (and we will turn back to him later) the commission employed the philanthropist Levin Christian Sander (1756–1819) in the year 1800. He came from the duchy of Holstein, then a part of the united Danish-Norwegian monarchy, and he was influenced of Kant's and Humboldt's ideas of Bildung and of Basedow's thoughts of pedagogy (Nordenbo 1980, Korsgaard [55] 2003, Korsgaard [35] 2004, Steffensen 1979).

Now a brilliant educational epoch started, and in 1809 the royal commission could end its long work and accomplish a reform of the Danish and Norwegian high schools such that Bildung was a central element and aim of the teaching. Unfortunately, however, this brilliant epoch became very short. Only a few students were matriculated at the new educational faculty as the Napoleonic Wars led to a total military disaster and defeat for Denmark. The double Danish-Norwegian monarchy was split up and Norway got its own free constitution and was forced into a union with Sweden in 1814, and already in 1813 the Danish state went bankrupt. The following deep economic crisis in the 1820'ies implied that the reforms of the high schools and the universities temporarily were given up.

THE IDEAL OF BILDUNG FROM KANT TO MADVIG

At the beginning of the 19th century and after Kant was dead, other philosophers treated the concept of Bildung. The most prominent of these philosophers was the famous German professor Georg Wilhelm Friedrich Hegel (1770–1831). He was an opponent to both Kant and Fichte, and he saw the educational development of the individual and of human culture and civilization as structurally coincident. Hence the individual was participating in a historical process of Bildung and the motive power of this process was "Anima Mundi" (The World Spirit). These considerations were outlines in his philosophical masterpiece "Phänomenologie des Geistes" (Hegel 1807), and the basic assumption of his philosophy is the claim: "Wahre ist das Ganze", Eng: "The truth is the whole". This means, according to Hegel, that we don't know the truth about the world until we know the whole truth and we have obtained "absolute knowledge". Hence to reach the aim of absolute knowledge we are participating in a forth running cultural process (Stjernfelt 2008).

Hegel's basic philosophical principles deal with the dialectic concepts "existence" and "non-existence" and the implication of these concepts "the synthesis". Hence Hegel's philosophy is holistic and dialectic. This has the following consequence: When an individual tries to perceive a concept (the thesis) the negation of this concept (the antithesis) will immediately come to the individual's mind, and the next step is to generate a synthesis of the first idea and its negation. Then this process can go on and on, in principle ad infinitum, and in this way human perception is lifted to a higher level (Stjernfelt 2008).

This dialectic philosophy has an interesting counterpart in modern quantum mechanics. An atomic particle is neither a classical particle nor a wave, but it obeys the principle of particle-wave dualism such that, depending on the circumstances, it might be observed either as a particle or as a wave. Hence, when a modern physicist thinks of an atomic particle (the thesis) he also has to think of it as a wave (the antithesis) and then the synthesis is the wave function associated to the given atomic particle. Particle and wave are complementary physical concepts. As the Danish physicist Niels Bohr (1885–1962) stressed this in the 1920'ies: "Our language is not sufficient to perceive the atomic world" and therefore new philosophical concepts, such as the particle wave dualism also called the "principle of complementarity", has to be taken into account in quantum physics. This is an interesting point of view that must be included as a central point of modern scientific Bildung.

As a reaction against Hegel's dialectic philosophy of Bildung the German psychologist Johann Friedrich Herbart (1776–1841) developed a new theory of Bildung and education in which didactics and teaching methods are included. He invented the concept of "Bildsamkeit", i. e. the ability for a human being to be educated and brought up (Nordenbo 1980, Koch [34] 2004). We hereby see that Herbart was significantly influenced of Kant's philosophy, cf. Kant's definition of being an enlightened individual.

And at the same time a new very critical debate about education and teaching at Danish high schools and at the University of Copenhagen broke out. Reforms of the educational system were absolutely necessary if the Danish society should be able to meet all the new challenges that came from the fast social and scientific development that took place in most European countries. Especially it seemed essential to include the natural sciences in the high school teaching and give up Latin as the main subject. Latin was a "dead" language and instead the students had to learn about different other topics that belonged to the real world, such as physics and chemistry. This point of view was put forward by many prominent researchers and teachers but especially Christian Lütken (1791–1856), who was a lecturer at the Academy of Soroe, emphasized in 1830 that the high school teaching was old fashioned and that reforms had to be carried out quickly (Krarup 1955, Nordenbo 1980). He was a warm adherent of the new ideas of Bildung, and he stressed that the students at the high schools would obtain insight and Bildung if they were taught natural sciences as well as if they were taught Greek and Latin. Shortly afterwards two highly respected professors from the University of Copenhagen, the physicist Hans Christian Oersted (1777–1851) and the philologist Johan Nicolai Madvig (1804–1886), began working for a new Danish educational system characterized by the concept of Bildung and the new-humanistic philosophy (Krarup 1995, Christensen [6] 2009).

H. C. OERSTED AND BILDUNG

The most famous Danish natural scientist in the first half of the 19th century was the physicist Hans Christian Oersted (1777–1851). In 1797 he graduated as a pharmacist from the University of Copenhagen and during the following year he travelled around in Germany and France where he got a lot of inspiration from many other skilled scholars. Especially he was influenced by the new romantic philosophy and movement, and he got a holistic interpretation of the natural sciences. Hence he believed that beyond the natural laws there was a higher idea and unification and he wanted to find this idea which he called "Spirit in Nature", Danish "Aanden I Naturen", (Christensen [5] and [6] 2009, Koch [33] 2003, Koch [34] 2004, Phil 1983, Oersted 1850).

H. C. Oersted was a warm adherent of Kant and his philosophy and he adopted his point of view regards Bildung and educational issues. He was also influenced by Fichte's ideas about lingual impact for human perception. Furthermore he was very interested in poetry, art, and other humanities and he believed that there existed a close relationship between art, language and the natural sciences. Here it is interesting to notice that the Saxon physician and painter Carl Gustav Carus (1789–1869) had quite similar holistic and romantic ideas: "Kunst als Gipfel der Wissenschaft", Eng. "Art as Culmination of Science" (Kuhlmann-Hodick 2009). To H. C. Oersted it was very important to find aesthetic features and patterns in nature and he investigated the so-called "figures of sound" that occurred when a thin metal plate covered with fine grain was put into oscillations using a violin bow. He discovered that these figures of sound were hypobolas, and this fact was a confirmation of his ideas about art and nature (Christensen [4] 2000, Christensen [6] 2009, Jackson 2000).

Such aesthetic considerations had already previously been emphasized by some mathematicians who had derived especially beautiful formulas. The most famous example is the formula $e^{i\pi} = -1$, where the three fundamental mathematical constants e (from analysis), i (from algebra) and π (from geometry) are brought together in a very simple relation with the value–1 (Sandifer 2007). The formula was found in 1748 by the Swiss mathematician Leonhard Euler (1707–1783) who, at that time, was working in Berlin, and American

physicist Richard P. Feynman (1918–1988) has called it "the most remarkable formula in mathematics".

In 1806 H. C. Oersted was appointed professor of physics at the University of Copenhagen, and in July 1820 he made his greatest discovery: Electromagnetism. In his laboratory he showed that an electric current was generating a magnetic field and immediately he became world famous (Christensen [5] and [6] 2009). In his own opinion he had discovered one single feature of "Unity of Nature" and this was a proof of the existence of "Spirit of Nature". In 1824 he made another great discovery when he was the first scientist to isolate the chemical element aluminum (Christensen [5] and [6] 2009).

Having done these great discoveries he wanted to strengthen the teaching in the natural sciences both at Danish high schools and at the university. He founded the "Society for Propagation of Natural Sciences", such that especially young academics could be taught in physics and chemistry using the modern teaching methods inspired of Kant's and Humboldt's ideas of Bildung. Furthermore, in 1829 he established the College of Engineering (Polyteknisk Læreanstalt), today's Technical University of Denmark, such that it became possible to study the practical and useful applications of physics and chemistry at a high academic level. His model of this new technical college was the French École Polytechnique but the pedagogical ideas were inspired from Kant and to some extent from the philanthropic movement as well. By the way it is interesting to notice that Richard P. Feynman (mentioned shortly above) was one of those scientists of the 20th century who knew how important Bildung and teaching physics must be related if educating young people should turn out being a success. At Caltech he was asked to improve the teaching on physics for bachelor students, and then he worked out "The Feynman Lectures on Physics" which became world famous and today he is considered one of the greatest university teachers ever. Feynman was later awarded the Oersted Medal (Danish: Ørsted Medaljen) of which he, fully rightly, was very proud.

H. C. Oersted's great interest in language, poetry, and art implied that he adapted the trinity of Bildung from Kant but he expressed it in his own very personal way: The True, the Good, and the Beautiful. In April 1833 he wrote his interpretation of this trinity in a book to the famous fairy tale writer, Hans Christian Andersen: The Reason in the Reason = the True, The Reason in the Will = the Good, The Reason in the Imagination = the Beautiful (Grum-Schwensen 2000, Christensen [5] and [6] 2009). And what Bildung, in his opinion, really is, he has put in the following short and poetic way: "Bildung is the stamp of reason" (Damberg et al 2006). Later, the Swedish writer, feminist, and educationalist, Ellen Key (1849–1926) has defined Bildung as "what is left in your mind when you have forgotten what you have learnt" (Stjernfelt 2008). Actually, her pragmatic definition is very close to Oersted's statement.

The great breakthrough of Bildung in Danish high school and university teaching was prepared and accomplished by H. C. Oersted and another famous professor at the University of Copenhagen: The philologist and politician Johan Nicolai Madvig (1804–1886).

MADVIG AND BILDUNG

Johan Nicolai Madvig was born in Svaneke on the island of Bornholm in the Baltic Sea and at the age of 13 years he was sent to the royal High School of Frederiksborg (Danish: Frederiksborg lærde Skole) in Hilleroed north of Copenhagen. He was not impressed of his teachers except two of them: Frederik Peter Jacob Dahl (1788–1864) who taught literature

and music and the rector, Bendt Bentsen (1763–1830), who was a philologist and to some extent influenced of the new-humanistic ideas. Especially Bendt Bendtsen, who had studied Greek and Latin in Göttingen during the years 1787–1789 where professor Heyne had been his teacher, was admired by Madvig, and no doubt he inspired the young student so much, that when Madvig in 1820 was matriculated at the University of Copenhagen he chose to study classical philology (Krarup 1955, Boserup 1992, Petersen 1993, Christoffersen 2005).

25 years old, in 1828, Johan Nicolai Madvig was appointed as professor of classical philology at the University of Copenhagen and soon he became one of the leading philologists of whole Europe. His research was as remarkable as Oersted's was and he was internationally acknowledged. Madvig was strongly influenced by the new-humanistic philosophical school and he supported and joined Kant's and Humboldt's educational ideas of Bildung and teaching. Just as H. C. Oersted did, Madvig wanted to modernize the Danish educational system, especially the teaching and the syllabuses of the high schools and the university had to be changed considerably (Krarup 1955, Petersen 1993, Christensen [5] and [6] 2009).

In Madvig's opinion the teaching of the high schools should no longer be based upon Latin. Although he was a famous philologist he thought that Latin, being a "dead" language, should not fill so much as it really did in the high school teaching. European culture, philosophy and science had their origin in ancient Greece and hence it was important teaching Greek culture and philosophy from the classical epoch. Of course, the students should know about Greek language, and also know about Latin to some extent, but the main purpose was knowledge about the culture that was the basis of European civilization and enlightenment. Furthermore, he also wanted most of the basic first year university courses to be transferred to the high schools and he wanted a unified high school teaching where pedagogical Bildung was extremely central. In this way the high school teaching would be brought to an essentially higher level, and at the university it would be possible to study more concentrated and to become more absorbed in the different sciences. The general academic level would be increased considerably and in all respects be influenced of Bildung (Krarup 1955, Korsgaard [55] 2003, Korsgaard [35] 2004, Damberg 2006, Christensen [6] 2009). This point of view was fully supported by H. C. Oersted (Christensen [6] 2009).

In the academic world Madvig obtained a very central position. During six periods he was elected rector of the University of Copenhagen and he also got an influential political position after the democratic breakthrough in 1848. He was elected to the new Danish parliament and he was appointed minister of education in November 1848. Therefore, Madvig had extremely good academic and political prerequisites of reforming the Danish high schools and the university, and he did it (Krarup 1955).

In 1848 a new study of economics was established at the university, such that teaching the social sciences were strengthened. In 1850 mathematics and the natural sciences were organized in a new independent faculty (Phil 1983) and a first year course called "Philosophicum", common for all university students, was established (Witt-Hansen 1970). "Philosophicum" was a course on the philosophical disciplines: Logics, psychology, and history of philosophy. Other philosophical topics were now transformed to the high schools and furthermore the high school teaching was modernizes, such that the natural sciences and Greek culture became a considerable part of the high school syllabus (Krarup 1955). All this was carried out such that the teaching was in full accordance with the ideas of Bildung, and the Madvig's educational reforms were approved by H. C. Oersted. The didactic revolution

led by Kant, Humboldt and several other philosophers had now been accomplished in Denmark, and interestingly enough similar reforms were accomplished in Norway.

BILDUNG AND SPECIALIZATION

In the second half of the 19th century the scientific development accelerated more and more, especially within mathematics, physics, and other natural sciences, but also other academic issues changed from purely philosophical disciplines to experimental sciences. Hence it became more and more difficult to maintain the ideal that high school teaching must be common for all students such that academic specialization didn't take place until the first year courses at the university. For Madvig this fact was a problem. He still wanted a unified high school teaching where all students had one and the same syllabus quite in Humboldt's spirit (Krarup 1955, Damberg 2006), but this point a view could not be maintained and Madvig realized that he had to accept new reforms of the educational system. Then, in 1871, the necessary legislation of these reforms was prepared and accomplished by the minister of education, the highly respected conservative national-liberal statesman Carl Christian Hall (1812–1888), and the bill was passed soon after (Krarup 1955).

Now, the high school teaching was split up into two different study lines: One historical-lingual line where the main subject were Greek, Latin and history and another mathematical line with the main subjects: mathematics, physics and chemistry. But the didactic and pedagogical methods were still based on the idea of Bildung. However, now Bildung had got two faces: A humanistic and a natural scientific (Krarup 1955, Damberg 2006). And, also in 1871, at the University of Copenhagen the first year course "Philosophicum" was shortened, but still it was a course common and compulsory for all university students (Witt-Hansen 1970).

All this was a consequence of the general scientific development. Since Humboldt established his university in Berlin in 1810 many things had happened and specialization both in high school and university teaching was absolutely necessary. Humboldt's and Kant's classic ideals of Bildung and education had to be adjusted little by little to the new academic demands and hence Bildung changed from one single concept to a diversity of different concepts depending on humanities or natural sciences. This was a development that took place in all European countries and in America too but the ideal of academic freedom and belief of teaching and research as inseparable issues were unchanged (Thomas Huxley [22] 1876, Helmholtz 1877).

In 1903 the next reform of the Danish high schools was accomplished. This reform was, in almost all details, inspired of a similar reform that had been accomplished in 1896 in Norway, and now the students could choose one of three different study lines. Modern living languages, such as English, French and German, became an essential part of the syllabus of one of the study lines, the "Modern Lingual Line". The two other lines "The Classical Lingual Line" and "The Mathematical Line" were almost unchanged from the well known study lines of the reform of 1871. It is also interesting to notice that a few years later infinitesimal calculus was introduced for the first time to the Danish high school students (Palle Bak Petersen et al. [53] 2003). But still, the didactic principles and the educational aims were based upon the traditional classical ideas of Bildung (Slagstad et al 2003, Damberg 2006).

Apart from several small adjustments this reform of 1903 was almost unchanged until 1963. During the "Cold War" it became more and more visible that western countries had to

intensify and improve the teaching of mathematics, physics and engineering at high schools and universities, if the Soviet Union shouldn't win the "weapon race". Especially after 4 October 1957, when the first Soviet satellite, "Sputnik I", began orbiting the Earth, it was absolutely clear to most politicians that something radically had to be done. In Denmark the following considerations were coined out into a decision that modern mathematics at a high abstract level (abstract set theory, classical analysis, abstract algebra, and group theory) should be taught to all high school students in the mathematical line, that modern advanced physical theories, such as elements of the special theory of relativity and Bohr's quantum model of the hydrogen atom, had to be a central and compulsory part of all physics courses at Danish high schools, and that aspects of Bildung must be integrated into all high school subjects. This was very ambitious and in 1963 this decision was carried out and a new reform of Danish high school teaching was accomplished (Slagstad et al 2003, Damberg 2006). At the same time the teaching of mathematics, physics and engineering was strengthened considerably at all the Danish universities. And, as something quite natural, Bildung played an important role in all university courses. Now, the Danish educational system had reached its "Golden Age".

Several university teachers did a great work building up courses of a very high level and their engaged teaching was always influenced of the ideas going back to Kant, Humboldt, Oersted, and Madvig. Among these teachers especially one became known for his great enthusiasm: The physicist Jens Martin Knudsen (1930–2005) who later became a very famous philosopher and astrophysicist (Olesen [49] 2009).

THE DECLINE OF BILDUNG

In July 1969 the first human being was standing on the Moon's surface. President John F. Kennedy's clear decision from his famous speech in May 1961, that "before the end of this decade (the 1960'ies) the United States should put a man on the Moon and bring him safely back to the Earth" had been crowned with great success. Now, everybody realized that the western world was leading technologically. Hence the United States was superior to the Soviet Union. Seen with American and Western European eyes this was a very good thing, and in Denmark and other western countries many people and especially many politicians began to relax. Now it wasn't so necessary as previously to teach advanced mathematics and physics to all high school students. Many more young people could pass high school teaching (and that became an important political aim), if these abstract theories were taken, at least partially, away from the syllabuses. Furthermore, several topics could be excluded from the syllabuses and instead of teaching according to classical Bildung ideas many new pedagogical experiments, based on quite other ideals, were launched (Slagstad et al. 2003, Damberg 2006).

Already in 1971 a new reform of the high school teaching was accomplished. The syllabuses of all subjects were reduced (with circa 20 per cent), the most difficult topics were excluded, and Klafki's modern ideas of "categorical Bildung" were interpreted in such an outrageous way that the educational authorities freely claimed that the Danish high school teaching was still based upon the classical theory of Bildung (Damberg 2006). [Wolfgang Klafki, German educational theorist born 1927 in Angerburg, East Prussia].

However, this claim was not true. The philosophical aspects of the different topics were definitely given up, and the academic level began rushing down. Also, in summer 1971, the first year (and previously compulsory) course, "Philosophicum", was cancelled such that there no longer were any common introductory courses for all university students. This was

indeed an evil attack on Humboldt's, Oersted's, and Madvig's educational and academic ideals, and it had the sad consequence that Bildung was no longer a well defined concept and educational aim, but just a word to be used arbitrarily when it was most convenient. The concept of Bildung was emptied and a much narrower specialization on an essentially lower level became the new political ideal (Damberg 2006).

And it got even worse. In 1988 a new reform of the Danish high school teaching was accomplished. In almost all subjects the syllabuses were reduced in a ridiculous way, such that several important issues were omitted. This had the consequence that for those high school students who wanted to continue their education at one of the universities some problems might occur when they started their studies. However, this was solved in a somewhat strange way by lowering the level at many first year studies. This was a bad development, also because the ideals of Bildung were pushed more and more away, but still no remarkable catastrophe had happened.

But now, in the 1990'ies, many more young people graduated from high school and many more students were matriculated at the universities. This implied that it became much more difficult to maintain the previous academic levels both in the high school teaching and at the universities. Bildung disappeared more and more. The elitist content of the higher education, which had been so important twenty years before, was totally swept away, and the aim was now mass production of students from the high schools and candidates from the universities. The Bologna process was implemented and the university studies were cut shorter from 6 to 5 years. Of course this implied that the candidates' qualifications decreased, and their knowledge was shrinking. Furthermore, a similar process took place in many other western states and also in the Scandinavian countries Norway and Sweden. Especially the natural sciences and mathematics were hit terribly.

Strange to say: This was not considered being a big political problem. Rather the contrary: Many politicians and educational authorities considered this development as a great progress. The ideals of Bildung and the traditional academic virtues were old fashioned, they said. Instead the new political aim was producing as many candidates as possible, no matter what qualifications they had got, and the educational institutions were run using management methods as if they were manufacturing companies. Especially after 2000 this management process became more and more implemented in university and high school administration (Slagstad et al. 2003).

The gap between the high school syllabuses and the introductory level at the universities grew bigger and bigger, and in 2005 new high school legislation passed such that this gap became even bigger. Although many educational authorities still talked a lot about Bildung, no Bildung was really left in high school teaching. Since Madvig's reform in 1850 the high school teaching had the main purpose preparing the students to start university or college studies. Now this noble aim was given up and instead the teaching was changed substantially and was split up of interdisciplinary sessions such that the compulsory syllabuses were reduced with circa 50 percent. Furthermore, in each subject the level of knowledge decreased so much that the interdisciplinary sessions were actually meaningless. Of the classical ideas of Bildung absolutely nothing was left.

To illustrate how horrible the situation has been in Danish high schools since 2005 we will look at the syllabus of mathematics. Before this last reform the students had to learn how to prove elementary propositions and they had to know the definition of several different

mathematical concepts. This gave them a satisfactory and quite good comprehension of mathematical reasoning and way of thinking. Now all this stopped. The high school students should only use big electronic calculators and computer programs when solving mathematical problems, but this was hopeless because they didn't understand what was actually going on. They had no definitions to deal with, they didn't prove any propositions, and they didn't know about mathematical method. In fact this sort of teaching was nothing worth. Pushing a botton doesn't give any perception or deeper knowledge, and in the same nonacademic way the teaching of most other subjects were also treated. In physics, for example, everything about electromagnetism and ideal gasses was abolished and Newton's laws were considered old fashioned, so why teach them?

This was modern and progressive teaching, the politicians and the authorities said, but in fact this was nothing but very old fashioned (and even pre-Socratic) teaching because the students were not taught how to analyze and how to solve problems and they were not taught anything that could give them some kind of academic competences and personal insight. So indeed, this was an educational tragedy.

And in the universities the study programs were changed, such that all courses only lasted one term of 14–15 weeks. The politicians required that the rates of failure and the drop-out rates at the universities must be reduced considerably. This was almost an impossible task to meet because the new students coming directly from high school had so poor and fragmented knowledge and no academic virtues. They felt, for natural reasons, that the university environment was totally strange for them, and fewer and fewer students were matriculated to university studies, especially in 2008 this was a national catastrophe.

Now, because the first year students had so poor knowledge from high school and no academic Bildung at all, the drop-out rate and the rates of failure increased, and for the universities this was a very definite and troublesome problem (Olesen [42] 2007, Olesen [43], [44], and [45] 2008, Olesen [46, [47], and [48] 2009). Of course it was very difficult to teach first year courses because you had to take up a lot of very simple items in order to span that extremely wide gap between the students' high school knowledge and basic level at the first year university courses. Clearly this problem was impossible to solve within the given frames. Something radical had to be done.

To argue for better high school teaching was waste of time. Hence there were two different possibilities left: 1. University institutes could choose decreasing the level of their first year courses further, but that would likely imply new serious problems for the subsequent university teaching, and the academic quality of the candidates might really be threatened. 2. Spending more resources on the basic first year teaching and introducing Bildung determinately. Then one might hopefully expect an important bridging to be established over the gap between the present high school teaching and the natural requirements of introductory university courses. Each university institute had to make a choice between these two very different options. Unfortunately, most of them chose option number 1—probably because that was the easier thing to do. But in autumn 2007 at the Department of Economics, University of Copenhagen, the other option was chosen with respect to the first year mathematics courses.

A HEURISTIC APPROACH

The first year teaching of mathematics at the Department of Economics is separated into two different courses, Mathematics A in the first term and Mathematics B in the second term. Of

course the students have to pass Mathematics A before they begin studying Mathematics B. In the years 2006–2009 the exams of Mathematics A were oral exams lasting 30 minutes, whilst the exams of Mathematics B were written. From 2010 the exams of Mathematics A are written with no aids allowed, and the exams of Mathematics B are still written.

During many years the rate of failure of the written exams of Mathematics B had been constantly in the interval from 15–25 per cent. Of course one must expect some more or less random fluctuations from year to year, and a rate of failure in this interval has always been considered acceptable.

In 2006 the rate of failure at the written exam of Mathematics B was 23.4 per cent. Hence this was in the interval of acceptance, but in 2007 the rate of failure suddenly increased dramatically to 34.7 per cent (Olesen [42] 2007). An analysis of the students' solutions showed that they had very poor elementary mathematical knowledge. They didn't know much about simple mathematical reasoning either, so now the strongly weakened high school teaching of mathematics with no elements of Bildung had been discovered as totally hopeless and of no use as a basis for first year university courses. The high school teaching didn't prepare the students for further studies anymore! This was a terrible fact beyond dispute. But how did we manage to change these things?

First of all it was very important to teach the first year students all that elementary mathematics they didn't know from high school. But to reach this clear aim and at the same time teaching all that mathematics that is in the first year university syllabus was absolutely impossible without more resources being allocated to the course. The lectures having been of two hours' duration each week so far had to be extended with no less than 50 per cent such that they became of three hours' duration. This was absolutely necessary from a pedagogical point of view such that the lecturer had enough of time to explain all details of the stuff so very profoundly that the students would feel personally secure and not be confused. This was done from spring term 2008.

Furthermore, to convince the students that mathematics is an academic discipline with many cultural aspects, also for students of economics, and to engage them in their studies and activate them, Bildung to some extent was introduced in the lectures. Some important historical aspects were taken up to explain how a given mathematical theory had been discovered, how it had been developed, and what scientific consequences it had given us. Also some interesting philosophical aspects of using mathematical models in an economical theory were discussed during the lectures, and in this way the teaching was guided of the intention of the very important aspect of Bildung: What does the (mathematical) knowledge I have acquired mean to me personally? This was one of Kant's points of Bildung (Olesen [45] 2008).

The classes, lasting three hours each week, were reorganized with a quite rigid structure, such that all students were obliged to accomplish some elementary mathematical proofs on the blackboard, at least twice during the term, and they had to work together in given groups such that they learnt how to help each other and to develop some social study competencies. Only one hour was left for the teaching assistant talking about some complex and difficult mathematical problems. Apparently this was very far from the classical ideals of the free university study, but anyhow it met Kant's aim of Bildung cultivating the individual as a responsible member of a community. In fact the students became citizens of their classes (Olesen [45] 2008).

Immediately it was quite obvious that most students became more engaged and more interested in mathematics and their study activity increased considerably. The teaching had become personally more meaningful to them and had given them some new important scientific perspectives. Not only encyclopedic knowledge was emphasized, but in a much larger extent categorical Bildung (cf. Wolfgang Klafki) influenced the spirit of the teaching. And at the exam of Mathematics B in June 2008 the rate of failure dropped to 25.0 per cent. This was a fully acceptable result and it was much better than result from June 2007. Apparently, the new way of teaching the first year university students had shown being fruitful (Olesen [45] 2008). Now the time has come to go further!

INTRODUCING MATHEMATICAL BILDUNG

In summer 2008 the intention was to strengthen mathematical Bildung as a tool in the first year university teaching for economics students. Therefore, in September 2008, the lectures started with the issue "What is mathematics and what is science"? Here the axiomatic method seen in a historical and cultural perspective as an important development of the human mind was immediately introduced and we concentrated about the following items: How did the Greeks in Antiquity create mathematics? What is an axiom? What is a proposition and what is a theorem? How do we use logic and how do we prove a proposition or a theorem only using what we consider to be true? Next the attention was turned to the most fundamental issues of epistemology and Kant's famous thoughts of scientific understanding, mathematical reasoning, and assertions being a priori, a posteriori, analytic or synthetic (Gowers 2008, Kant [27] 1787, Kant [32] 1804, Olesen [41] 2007, Olesen [48] 2009, Wolff 1963).

This seemed to be very difficult and very theoretical to a vast majority of the students because this was quite new to them, but they realized, that there exists a much larger and more interesting world of thought and scientific behavior than they had ever expected and soon they began showing a new personal academic engagement which is very important for all university students. They began participating in philosophical discussions and they were eager to learn much more. They had really been motivated to their university studies!

The lectures went on telling about mathematical formalism and abstract set theory starting with Euler's diagrams or circles from 1761 (Sandifer 2007) and George Boole's logical system (Boolean algebra) from 1854 (Gowers 2008, Wolff 1963), then Georg Cantor's theory of sets and a little about countable sets and transfinite cardinals (Gowers 2008), and after this ending up with Bertrand Russell's famous paradox from 1904 (Gowers 2008, Olesen [41] 2007). This was done to show the students that a logical system has certain limitations that might lead to strange and surprising paradoxes. This is a very important fact not only in mathematics, but in all academic disciplines. Hence it was quite natural to talk about Kurt Gödel's incompleteness theorems from 1931. In this way mathematical Bildung was broadened out as a general ingredient of university studies. Hence we suddenly came closer to Kant's and Humboldt's classic ideas about Bildung (Gowers 2008, Olesen [41] 2007, Olesen [48] 2009).

All this was also philosophical and very theoretical, but it was very important and inspiring for the students to know about scientific methods such that they gained new useful academic competences. But of course it was also very important and motivating to work with some apparently practical problems in a larger scientific context. Talking about an old given mathematical problem that was solved many years ago and later generalized to a modern theory with important applications within other disciplines was the main idea for selecting

relevant topics. Here the lectures first concentrated about describing the famous problem of passing each of the 7 bridges in the city of Königsberg only once in one continuous path. The problem was solved by Leonhard Euler in 1735 (such a path didn't exist!) and later on he developed his general method so much that it became possible to build up a useful theory for network analysis and advanced graph theory which is so important for modern computer science and economic theory (Hopkins [18] 2004, Hopkins [19] 2007, Olesen [41] 2007, Olesen [48] 2009, Wolff 1963). This narrative and many other narratives from the history and foundation of mathematics (often with close relations to economic theory) had a remarkable impact on the students' engagement and helped them looking at mathematics as an important part of human culture and science. In fact all these narratives became the core of what we might call "Mathematical Bildung", and now "Mathematical Bildung" was a pedagogical tool in the basic university teaching of the first year students at The Department of Economics (Olesen [48] 2009).

After this philosophical and historical introduction to the lectures of mathematics it was extremely important to continue this process using "Mathematical Bildung" in the subsequent lectures where many different specific topics were taught. For example: Lecturing on "orthogonal matrices", which were introduced by the French mathematician Charles Hermite in 1854, we went further back to 1770 when Euler for the first time considered a system of linear equations in which an orthogonal matrix was used implicitly without knowing anything about matrices in general or orthogonal matrices in particular (Sandifer 2007). This was done with the definite purpose that the students should see how mathematics as a deductive science has developed and such that they got an idea of how scientific research often is based upon solving a specific problem and then later on generalizing this to a proper theory (Olesen [48] 2009).

For the students, knowing almost nothing about mathematics and absolutely nothing about mathematics as a science when they were matriculated at the university, "Mathematical Bildung" was new, surprising, and challenging. They also became more interested in doing mathematics on their own and they became much more active and personally engaged. So what we had seen of positive signs since the first elements of Mathematical Bildung heuristically were introduced in spring 2008 was now seen much clearer. Therefore, it was very exciting to experience how the students' exams of Mathematics A in January 2009 would turn out.

The result was quite remarkable: 145 students were examined in "Mathematics A", and the rate of failure was only 9.7 per cent—the lowest rate ever seen at a first term exam of mathematics at the Department of Economics. This was indeed very encouraging!

Now it was time to put a very relevant question: Was Bildung really the main reason for this good result? The answer was in fact "Yes". But how could we know this?

At the Faculty of Pharmaceutical Sciences at the University of Copenhagen all first year students are forced to have a math-course during their first term. This course is very similar to the first term course "Mathematics A" at the Department of Economics. But at the Faculty of Pharmaceutical Sciences no further resources were allocated to enlarge the lectures with 50 per cent such that elementary mathematical methods (previously known from high school) and aspects of Bildung could be integrated into the teaching.

The rate of failure at the Faculty of Pharmaceutical Sciences had, during many years, not exceeded 15 per cent and often it was lower, but since 2005 it began increasing, and in January 2009 it reached 39.6 per cent (Olesen [46] and [48] 2009) where 207 pharmacy students participated in the first term exam of mathematics. This was a disaster, and it showed that new didactic methods must be brought into the first year university teaching if the students must pass their exams. So now we know for sure that teaching elementary mathematical reasoning and aspects of Bildung is a useful way to solve a big problem for Danish universities after the high school teaching has been destroyed totally. Maybe this is not the only way to limit the rate of failure and make the students more engaged and motivated, but in fact it is a possible way and a way that works. And it is not only a possible way that works at Danish universities because similar problems have occurred at universities in many other countries, especially in Norway and Sweden.

MODERN BILDUNG

Bildung is hardly time dependent, but the modern universities—both in Denmark and many other countries—are mass universities where the criterion of success is to produce as many candidates as fast as possible. The free Humboldt University as a classical university concept has disappeared, and it is unlikely it will come back. The government and the educational authorities are interested in minimizing the costs of educating a candidate and therefore no time must be wasted and in many incidents the academic level has been decreased—maybe not drastically but to some considerably extent that might hurt in some cases. Hence if we wouldn't accept a lower academic level, and that is the intention at the Department of Economics, we must use aspects of Bildung in our teaching—not only in mathematics but also in all other subjects, at least in the basic first year teaching. Since the free and classical Humboldt University doesn't exist anymore, Bildung must be adjusted to the given conditions we have at the beginning of the 21st century.

Such an adjustment was exactly what was developed since the first heuristic attempts began in autumn 2007 and now this development had to be continued. That was actually done during the next term in spring 2009, where the students had their next math course, "Mathematics B", which is more advanced and more abstract than "Mathematics A". Would it really be possible to obtain an even better result than that we had in June 2008, where the rate of failure had been 25.0 per cent? Yes it really was, and in June 2009 the rate of failure of "Mathematics B" dropped to only 12.7 per cent. No doubt: Bildung is a useful tool in basic university teaching.

Next academic year began in September 2009 and now the ingredient of Bildung in the first year teaching of mathematics at the Department of Economics was further increased. Not only general philosophy of science and many historical topics were lectured at the beginning of the first term course, "Mathematics A", but also some aspects of economic history were introduced. For example the French philosopher Francois Quesnay's famous theory from 1758 about a free economical system considered as a self-developing system according to certain laws, which provided the foundation of the ideas of physiocratism, were mentioned, and it was emphasized that these ideas also made the foundation of a mathematical description of a dynamical economic system, just as it is well known from physics. Furthermore the economic philosophers Robert Jacques Turgot (1727–1781) and Adam Smith (1723–1790) were mentioned and theirs ideas of economical physiocratism and liberalism respectively.

This was done to show to the students that it is convenient and necessary applying abstract mathematics in order to describe a self developing economical system, and then the time has come to introduce the so called "St. Petersburg Paradox" formulated by the Swiss mathematician Nicolaus Bernoulli (1687–1758) in 1713. This very difficult problem was solved in 1738 by his cousin, Daniel Bernoulli (1700–1782), using the concept of utility functions and the solution was shortly designed. In this way it became quite clear to the students that the apparently different disciplines of their first year studies were closely connected to each other. They got interdisciplinary academic Bildung, were more engaged in their studies from the very beginning, and they were strongly motivated.

Now the lectures were not only lectures of mathematics but also lectures with so many interdisciplinary aspects that Bildung had become a very important issue of teaching mathematics at the Department of Economics. The students obtained a certain and necessary amount of mathematical knowledge but they also saw mathematics as a topic in an interdisciplinary and broader perspective. In this way the lectures had, right from the beginning, given them something personally making them educated in a classic academic way.

In modern theory of Bildung one usually considers an equilateral triangle where one side represents knowledge (encyclopedic Bildung), insight and perception, the other represents reflection and judgment (formal Bildung), and the third represents communication of knowledge and relationship between a person and other people. This is the so-called "triangle of Bildung", and we notice that the lectures, as described above, satisfy the aims of the first and second side of this triangle. But what about the third side? The aims of this side were met in the classes. Here the group work was organized such that the students were given a schedule of each class and all the problems had to be solved by the groups before having class. Then there was time enough to the students' presentations of the solutions of the given problems, to discussions, and to prove different theorems on the blackboard. In this way a very important social aspect was introduced in the classes. The students had to work together in their groups, they had to help each other, and they had to communicate their knowledge to the other students (cf. Bohlin [2] 2010). Furthermore, some aspects of study technique also had to be a part of the classes, and in this way the students obtained both certain mathematical competences and some social and communicative competences.

The triangle of Bildung represents the modern concept of Bildung that can be practiced in today's university teaching. It is interesting to notice that this triangle also represents the classical concept of Bildung such as it was described by Danish physicist and philosopher Hans Christian Oersted. He was talking about the truth (science), the beautiful (aesthetics), and the good (general moral) as the three aspects of Bildung. The truth is analogous to our scientific knowledge, the beautiful is analogous to our personal reflections and judgments, and the good is analogous to our helpfulness and communicative competences. Here we should also remind ourselves of the "trinity of Bildung" and Kant's three critics: Critic of Pure Reason, Critic the Judgment, and Critic of Practical Reason. Hence, the well known classic concept of Bildung has now been given a modern promotion that fits to the mass university of the 21st century.

Now, it was very interesting to experience how the assessments of the students' written exams of "Mathematics A" turned out. In January 2010 the rate of failure of this exam dropped once again to only 8.7 percent. Bildung in a modern mass university, in many

respects far away from Humboldt's ideal university of 1810, really works 200 years later in 2010, and the students were enthusiastic.

CONCLUSION

In this paper we have noticed that the Danish high school teaching has become so weak and poor, especially since 2005, that the students don't learn either mathematical thinking or enough of elementary mathematics. Furthermore, we have seen that all the aspects of classical Bildung, which were introduced in Danish high school teaching by Johan Nicolai Madvig in 1850, have disappeared. Teaching university students in the 21st century was, seen from a didactic point of view, bombed more than 150 years back to that time where the first year university teaching was merely an introduction to scientific studies. But because we nowadays have a modern mass university with many students matriculated and because the educational authorities require that the students graduate much faster than they did just a few years ago, it is absolutely impossible to make the first year university teaching just introductory. Then, ether the academic level had to be decreased or the teaching had to be changed such that it became more efficient and engaged the students.

We have seen that the classic Bildung, known from Kant, Humboldt, Madvig, and Oersted, can be revitalized in a modern form (cf. the Bildung triangle) such that it becomes a useful tool in first year university teaching. It gives the students the academic challenges they need, it makes them engaged, helpful and active, and they learn much more and pass theirs exams with a considerably higher rate than they previously did. Also they become more satisfied about their studies and they obtain a high level of important academic Bildung. So this is a great success.

Next step is to make Bildung an important part of all subjects the students meet at their first year studies. When this has been accomplished they will probably see these different subjects and topics as a wide range of aspects of one greater unity: Modern economical theory. And they will consider modern economical theory not just as an important scientific discipline but also as a cultural and philosophical discipline.

Of course I will warmly recommend this way of university teaching to all studies at all universities throughout the whole world because academic Bildung brings people of all cultures together into a great community working for peace and enlightenment.

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